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Measurement, Modelling and Synthesis of Violin Vibrato Sounds

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Summary
We consider the influence of vibrato on the sound of a violin by direct measurement, theoretical modelling and synthesised vibrato tones. Detailed analysis of bowed vibrato sounds reveals complex, asymmetric with respect to time, periodic variations in frequency, amplitude and timbre. To account for such features, a dynamic model for vibrato is described, based on the multi-resonant dynamic response of the violin and performance space acoustic. The model is extended to simulate the vibrato tones of a violin in a chosen performing acoustic from a single measurement of the sound at the listener's position produced by a short impulse at the bridge, assuming a frequency modulated sawtooth bowing force at the bridge. Measurements, modelling and synthesis all underline the increasing importance of the performance acoustic on the vibrato-induced fluctuations of the violin's tone, as one moves away from the violin.

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1. Introduction

Vibrato involves the cyclic modulation of the frequency of a note played on a stringed instrument as the player gently rocks the finger stopping the vibrating length of string backwards and forwards. Violin vibrato rates are typically in the range from 4 to 6 Hz and are similar to those used by singers [1] and in the playing of many other musical instruments [2]. Typical frequency modulation widths used in violin playing can approach a quarter-tone [3]. In addition to the modulation in frequency, vibrato on the violin and other bowed stringed instruments results in large cyclic fluctuations in amplitude, as previously reported, notably by Fletcher *et al.* [2, 4], Matthews and Kohut [5].

The historic use of vibrato to enhance the 'warmth' and 'singing quality' of the violin tone is well documented (see [6] and [7] for references from Ganassi (1542) to Leopold Mozart (1756)). Initially, vibrato was used sparingly as a special effect and to mimic the sound of the singing voice. However, recordings suggest an increasing use of vibrato by the modern solo performer over the last century, so that today a rather large-amplitude fast vibrato is ubiquitous in many performances of the romantic repertoire. In 'authentic' performances of the baroque and early classical repertoire, violinists tend to use a rather smaller amplitude vibrato, but only in the most purist of performance is vibrato entirely lacking. In any subjective assessment of violin tone quality, it is therefore almost impossible to ignore the influence of vibrato.

In this paper, we will focus on the way in which vibrato affects the sound produced by the violin from a scientific point of view and leave a more extensive discussion of its influence on musical performance and the perception of violin tone quality for a publication elsewhere. Nevertheless, our measurements, theoretical modelling and computer simulations illustrate why vibrato has such a strong influence on the perceived sound of the violin and related stringed instruments. They also demonstrate the large role of the performance acoustic in modifying the sound of a note played with vibrato. This is clearly an important factor in any assessment of the 'intrinsic' sound quality of a violin played in the concert hall or recording studio.

Although most players and listeners intuitively identify vibrato with the relatively small cyclic variations in pitch, in practice, vibrato also induces very large fluctuations in the amplitude of the bowed note and its component partials. These fluctuations are easily understood in terms of the cyclic scanning in frequency of the bowing force at the bridge and all its partials across the highly peaked, multi-resonant, acoustic response of the instrument. As a result, vibrato leads to complex waveforms with cyclic modulations in frequency, amplitude and spectral content or timbre within each period of the applied vibrato. This was recognised as a key feature of the violin vibrato sound by Meyer [3] and earlier by Fletcher and co-researchers [2, 4] and Mathews and Kohut [8]. These authors highlighted the likely role of vibrato in enhancing the sound of the solo violin above that of any accompanying players.

Psycho-acoustic test have shown that, for typical vibrato frequencies used in musical performance, the ear is much more sensitive to fluctuations in amplitude than to fluctuations in frequency or, more strictly, phase modulation –

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by a factor of around ten at 6 Hz (Sek [9]), as discussed by Moore[10]. Whereas the ear can easily perceive the changes in pitch of a note for vibrato rates below ∼3 Hz, for the typical vibrato rates used in violin playing (∼4–6 Hz), the perceived sound is of a pulsating tone at the mean frequency, which Prame [11] refers to as vibrato of the second-kind. This is illustrated by comparison of the similar sounds of a synthesised 1% frequency modulated and a 35% amplitude 1 kHz vibrato tone, SOUND 1, in the downloadable Powerpoint presentation [12], which will be used to demonstrate almost all the sounds discussed in this paper.

A common technique to determine the temporal variation in pitch and amplitude of waveforms has been to use time-shifted, short period, Fast Fourier Transforms (FFTs). Such information can be presented as a 3-D spectrogram illustrating the temporal variations in the amplitudes and frequencies of the individual partials of the bowed violin vibrato tone (see, for example, [3]). Such methods produce a discrete, time-averaged, frequency spectrum, with a resolution $\Delta f = 1/T_s$ inversely proportional to the sampled length T_s. Other methods, using more sophisticated computer algorithms, have been developed to follow temporal pitch changes more precisely. For violin tones, these include computer-based phase-vocoder measurements by McAdams, Beauchamp and Meneguzzi [13], cross-correlation techniques by Ando and Yamaguchi [14], modal distribution function analysis by Mellody and Wakefield [15], and wavelet analysis by Alm and Walker [16].

In such studies, Mellody and Wakefield showed that the realism of re-synthesised violin vibrato sounds was almost entirely associated with the fluctuations in amplitude of the partials rather than the frequency modulation, which had relatively little effect on the perceived sound quality. They also noted that bow-noise [17] added realism to violin sounds, as previously suggested by Fletcher and Sanders [2]. For simplicity, we will ignore such complications in this paper.

When vibrato sounds were recorded close to the violin, Meyer [3] observed a quasi-sinusoidal variation of the pitch of the bowed note, but with large fluctuations in the amplitude of component partials typically in the range of 3–15 dB, but sometimes exceeding 25 dB. However, at a distance, the fluctuations of both amplitude and frequency increased in complexity, with the derived frequencies fluctuating almost at random over the width of the vibrato- broadened partials. Such effects were attributed to contributions from the direct and reflected, time-delayed and hence frequency-shifted, frequency modulated sounds from the walls of the performance space. However, no detailed analysis of the resulting vibrato tones was attempted. Meyer suggested that the spreading in energy density of the reflected waves over the spectrum would reduce local saturation effects on the basilar membrane and thereby significantly enhance the perceived loudness and projection of the violin in the concert hall, as previously proposed by Fletcher and co-researchers [2, 4] and Matthews and Kohut [5]

The purpose of this paper is to explore the nature of vibrato tones in realistic performance and recording spaces, to develop a dynamic model for vibrato to describe such tones, and to apply the model to synthesise vibrato tones based on the impulsive response of the violin recorded at the listener's position in the performing space.

The paper is structured as follows. In the next section, we present a detailed analysis of a selected vibrato tone played on an exceptionally fine Stradivari violin by an international artist and of the same note played on a Vuillaume violin by the author. These measurements reveal a number of previously unreported features, which any theoretical model for vibrato must describe. In section 3, we account for such features by a dynamic model for vibrato involving the transient response of both the violin and performance acoustic. In section 4, we extend our model to the simulation of vibrato sounds based on the measured tap-tone response of the violin in a given performance space, assuming the force at the bridge can be simulated by a sawtooth waveform considered as a sequence of Helmholtz step-functions superimposed on an acoustically unimportant linear ramp. In section 5, we discuss the relevance of this research to the perception of vibrato tones and any assessment of violin quality. We also consider how the quality of the wood used in the construction of the violin, the additional damping introduced by holding the instrument, and the resonances of the undamped strings affect the vibrato-induced fluctuations and hence quality of the sound of the bowed note played with vibrato. Our finding are briefly summarised in section 6.

2. Violin vibrato tones

To illustrate the influence of vibrato on the sound of a violin, we first present a detailed analysis of the waveforms, envelopes and zero-crossing, inverse-period, ‘frequencies’ of the first five partials of two sampled D4 notes played with vibrato on the G-string.

The first example is taken from a BBC interview, in which the concert violinist Tasmin Little demonstrates the outstanding quality of a Stradivari violin (believed to be the “ex-Goldman”) previously played by Milstein, one of the greatest violinists of the last century. Figure 1 shows a 1.5 s section of the waveform envelope recorded at 44.1 kHz, an FFT of the whole waveform illustrating the vibrato-induced widths of the individual partials, a 50 ms section of the cyclically changing waveform, and two short-period FFTs (1024 points, Hanning-windowed) recorded at intervals 50 ms apart illustrating the cyclic changes in the spectrum and hence timbre of the perceived sound.

This example is of special interest, because the player repeatedly returns to this particular note to exemplify the outstanding quality of the instrument, describing it as “fantastically exciting, very vibrant, deafening and alive with a spirit that is absolutely desperate to get out”. Since, we continue to search for physical attributes of a violin's tone that correlate with the subjective judgement of violin tone by player and listener, there is obvious merit in analysing a sound considered to be of outstanding quality and played by an international artist.

Figure 1 illustrates the very large, vibrato-induced, quasi-periodic variations of the envelope, wave-form and spectral content of the recorded sound with time. We have deliberately chosen to concentrate on the sustained tone of the instrument and ignore the initial transients, which are widely recognised to be important in the initial identification of any instrument. Nevertheless, as in any recording of a sustained note on any violin, the recorded sound is immediately recognisable as that of a violin, wherever the "play-back" is started within the envelope. In contrast, if a single period of the recorded waveform is repeated continuously, the sound is indistinguishable from that of a crude electronic synthesiser (SOUND 2 [12]) and reference [18]. These examples demonstrate that fluctuations in the waveform are essential for any realistic simulation of violin tone.

By far the largest fluctuations are the vibrato-induced fluctuations in amplitude. However, additional fluctuations in amplitude in frequency including those from variable bow pressure and stochastic bow noise [17] may also be significant – particularly for the sound of open strings played without vibrato. Throughout this paper, we focus on the effect of vibrato on long notes, while recognising that the initial transients may by just as important in identifying and defining the quality of an instrument.

Figure 2 shows amplitude and inverse-period "frequency" fluctuations of each of the first five partials extracted from the recorded sound using software-implemented FFT filters (SOUND 3 [12]). The partials were extracted from the waveform by band-pass filters centred on the mean frequency of the n-th partial with pass-band widths of $\pm 20 \times n$ Hz. Doubling the filter width had no significant affect on the fluctuations in amplitude of the partials. The amplitudes of the 1st and 4th partials have been divided and multiplied by a factor 2 respectively for graphical convenience.

From Figure 1, we note that vibrato significantly broadens the width of the partials, so that the higher partials span an increasingly large fraction of the instrument's spectral range, particularly in the perceptually important frequencies above 1 kHz. Figure 2 shows that the cyclic modulation in amplitude of the partials can sometimes be as large as 100%, with envelope reminiscent of beating between sine waves of similar frequencies and amplitudes. The modulation is also strongly asymmetric with respect to time, which is a clear signature that the observed effects involve dynamic processes.

Because of the filtering, the waveforms of the individual partials are far less complicated than those of the overall waveform. One can therefore define and measure the "instantaneous frequency" of the individual partials from waveform zero-crossing events. Such measurements are illustrated for the first five partials. Only for the fourth partial can the modulation in frequency be described as even approximately sinusoidal, with a fractional modulation width of order 10 Hz in 290 Hz, about a quarter-tone. In contrast, the zero-crossing frequencies of the other partials, although showing an underlying cyclic modulation, exhibit many anomalous features, which are strongly correlated with sudden changes in slope of the waveform envelope. A specific example is illustrated in the top-right trace of Figure 2, for the first partial when the amplitude of the envelope is close to zero.

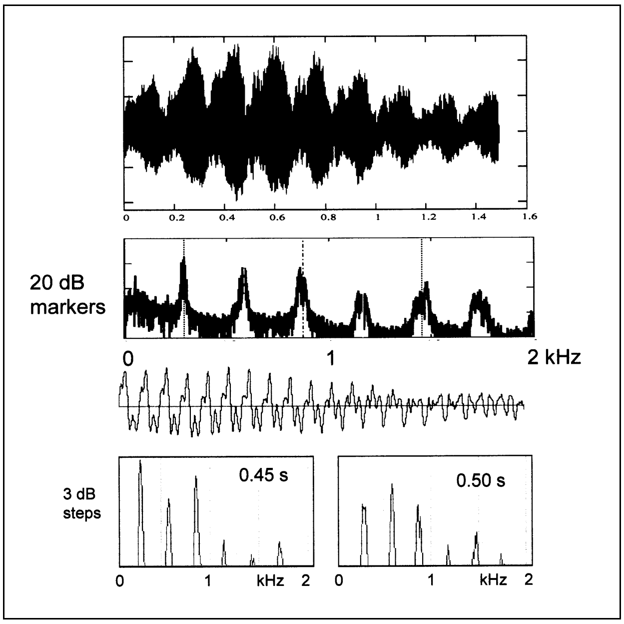


Figure 1. The amplitude and FFT spectrum of a 1.5 s section of the note D4 played with vibrato on a Stradivari violin, with a short section of the cyclically varying waveform and two short period FFTs spaced 50 ms apart showing the changes in spectral timbre with time.

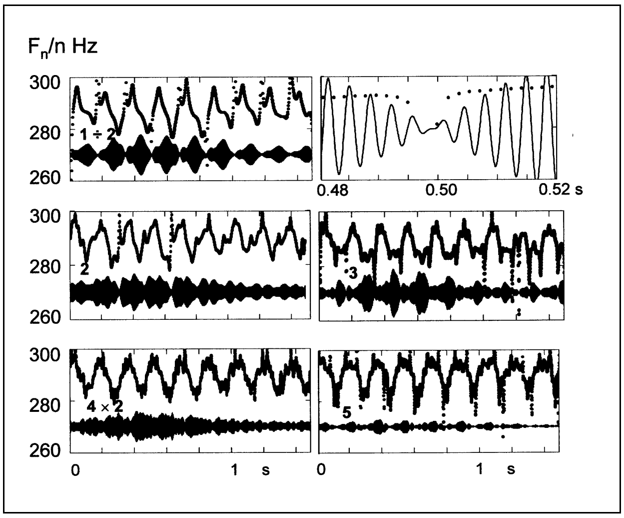


Figure 2. Temporal fluctuations of the normalised instantaneous, inverse-period, "frequency" and the amplitudes of the first five partials of the D4 vibrato tone played with a wide vibrato on a Stradivari violin.

The original sound and extracted sound of the first five partials are illustrated SOUND 3 [12]. Each of the broadened partials, when sounded separately, sounds like a note of constant average pitch pulsating at the vibrato frequency of ∼6 Hz.

The above effects cannot be described by simplistic models of vibrato, which relate the changes in amplitude of the various partials to the quasi-static cyclic frequency modulation of the multi-resonant response of the violin. Although such models predict large changes in amplitude of the individual partials, they cannot account for the strongly asymmetric fluctuations observed and envelopes that pass through zero. We will show that such features are characteristic features of the dynamic response of the instrument and performance space to the frequency modulated sawtooth bowing force.

Meyer [3] showed that the sound at a distance from a violin played with vibrato is strongly influenced by the time-delayed, and hence frequency-shifted, echoes from the walls of the performance space. To illustrate this effect, we first recorded the “close-field” vibrato sound of a French violin by Vuillaume and then used it as an isotropic source in a computer-simulated, relatively reverberant, performance acoustic. The close-field sound was recorded a few centimetres above the top-plate using a small electret microphone attached to the front of the chin rest. Close to the violin, the recorded sound is dominated by the sound of the violin rather than the acoustic environment. The FFT of the recorded note is shown in Figure 3 followed by the first 3 seconds of the envelope and inverse-period frequencies of the first 5 partials (SOUND 4 in reference [12]).

In contrast to the previous example, the envelopes of the inverse period frequencies of the partials of the Vuillaume violin recorded close to the instrument are much more nearly sinusoidal, with the amplitudes of the partials varying far less dramatically. Nevertheless, the fluctuations of the higher partials already show marked deviations from time-reversal symmetry. The player-defined vibrato modulation frequency of around 4.5 Hz is significantly less than the previous example as also is the modulation width of about ±3 Hz.

To illustrate the influence of the performing space acoustic on the sound of notes played with vibrato, we used the echo-chamber facility of the audio-processing CoolEdit software [19]. The sound of the closely-recorded violin was used as an isotropic point-source, at a distance of 8 m from the recording microphone (or listener), in a rectangular box-shaped room of dimensions $10\times10\times8\,\mathrm{m}^3$, with all surfaces reflecting 0.9 of the incident sound, corresponding to a Sabine 60 dB reverberation time of ~ 2.2 s. The listener and violin were placed off-axis at a distance of 1m from opposite end walls to couple to both symmetric and asymmetric room modes. Although the detailed features of the fluctuations in amplitude of the contributing partials are strongly dependent on the positions of the performer and listener in the performing space, the qualitative features (i.e. asymmetric waveform envelopes and partials with amplitudes that often pass through zero) are relatively insensitive to the exact position of violin and microphone.

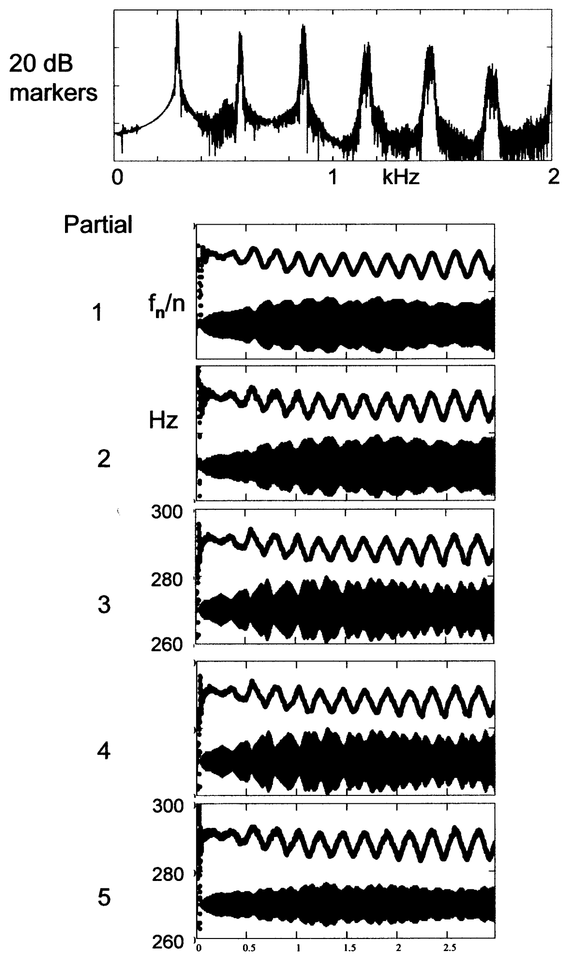


Figure 3. The FFT spectrum of the first 3 s of the recorded vibrato note D4 on a Vuillaume violin recorded close to the top plate followed by the amplitudes and inverse period frequencies of the first five partials.

Figure 4 illustrates the dramatic increase in the complexity and size of the amplitude modulation of the partials arising from the time-delayed, frequency-shifted, reflections from the walls of the performance space (SOUND 5 [12]). For the higher partials in particular, the fluctuations in frequency no longer vary even approximately as simple sinusoids, with waveform envelopes that are strongly asymmetrical with respect to time and frequently pass through zero (e.g. partials 3 and 4). Although the player-defined magnitude and frequency of the vibrato-induced fluctuations are less than those used in the Stradivari violin example, the qualitative features of the vibrato sounds are very similar, as indeed are the sounds of any violin played with vibrato in any resonant acoustic.

3. A dynamic model for violin vibrato

To describe the observed asymmetric temporal variations of the waveforms, we develop a dynamic model for vi-

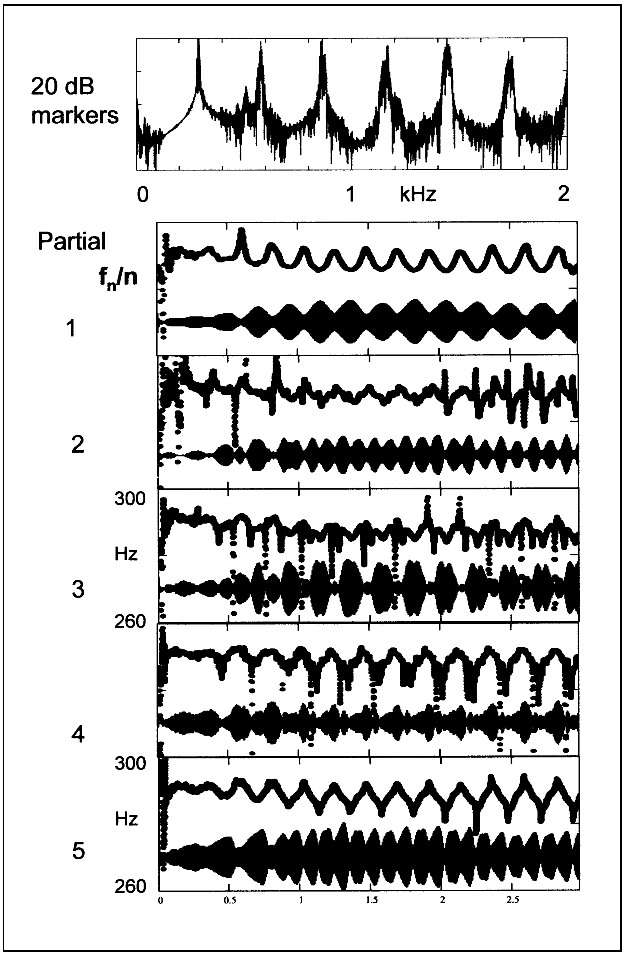


Figure 4. The FFT spectrum of the waveform of the closely-recorded Vuillaume violin vibrato D4 tone “played” in a 10× 108 m^3 performing space with a Sabine decay time $T_{60\,\mathrm{dB}} \sim 2.2$ s, illustrating the increased “complexity” in both amplitude and “frequency” fluctuations and increased departures from time-reversal symmetry in a reverberant performing acoustic.

brato, which involves the dynamic response of both the violin and the performing acoustic. The model accounts for many of the features of the vibrato tones described in the previous section.

We assume that vibrato results in a frequency modulated periodic bowing force on the bridge with phase-modulated frequency components varying as

$$F(\omega_n) = A\cos(\omega_n t + a\sin\Omega t), \tag{1}$$

where Ω is the vibrato frequency and a is the modulation parameter. This corresponds to a cyclic modulation of the frequency components,

$$\omega_n' = \omega_n\left(1 + a\frac{\Omega}{\omega_n}\cos\left(\Omega t\right)\right), \tag{2}$$

with a fractional modulation in frequency of $\pm a\Omega/\omega_n$. Small amplitude frequency modulation ($a \ll 1$) results in two side-bands of relative amplitude $a/2$ shifted by $\pm\Omega$ from and in phase-quadrature with the central unshifted frequency component. However, for typical fractional frequency modulation widths of a few percent, a is typically of order unity (e.g. for our first example, $\Delta f/f \sim \pm 5/290$, so that $a = 1$ for a vibrato rate of 5 Hz). The Fourier spectrum of a frequency-modulated sine wave then has a series of side bands at $\pm p\Omega$, where p is an integer. Figure 5 shows the spectra of a 1.5 second 300 Hz sine-wave modulated at 5 Hz for typical values of a for vibrato notes on a stringed instrument.

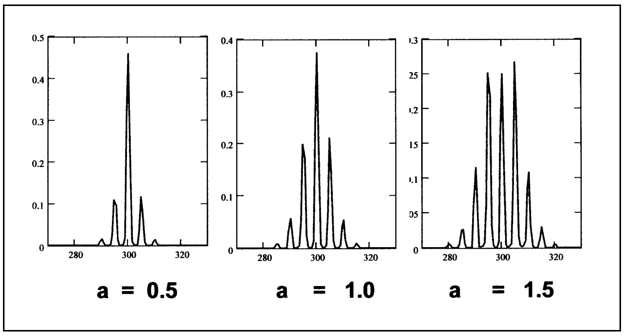


Figure 5. FFT spectra of a 300 Hz sine wave, phase-modulated at 6 Hz, for typical vibrato- induced modulation parameter a.

The excitation of vibrato-induced frequency- and phase-shifted side-bands, which can be comparable in size to the central components, results in strong interference and beating effects, particularly from multiple, time-delayed, reflections from the surfaces of the performance space.

If we assume that the response of the violin is linear with velocity-controlled damping, the steady-state radiated sound pressure $p(r,\theta,\phi,\omega)$ for a sinusoidal driving force $F(\omega)$ at the bridge can be written as

$$p(r,\theta,\phi,\omega) = \sum_j \frac{C_j}{\omega_j^2 - \omega^2 + \mathrm{i}\omega_j\omega/Q_j} \cdot R_j(r,\theta,\phi,\omega)F(\omega). \tag{3}$$

In this expression, ω_j and Q_j are the resonant frequencies and quality-factors of the damped normal modes of the instrument, which take into account the coupled motions of all the component parts (i.e. the body, front and back plates, neck, fingerboard, tailpiece, strings, bridge, etc), C_j is the amplitude of the j-th mode excited by the localised force $F(\omega)$ of the bowed string at the bridge, while $R_n(r,\theta,\phi,\omega)$ describes the radiated sound field of the excited modes as a function of distance r, polar angles $\theta$and ϕ and angular frequency ω. $R_n(r,\theta,\phi,\omega)$ will involve resonant contributions from all the acoustic modes of the performance space. Strictly speaking, the response should be described in terms of the normal modes of the coupled violin and surrounding acoustic, but we assume such coupling to be sufficiently weak to consider them separately.

For a bowed violin string, $F(\omega)$ can be approximated by a sawtooth waveform at the bridge, with Fourier components inversely proportional to the order n of the excited partials. The radiated sound will therefore have Fourier components at the same harmonic frequencies, but with amplitudes and phases varying markedly with the frequen-

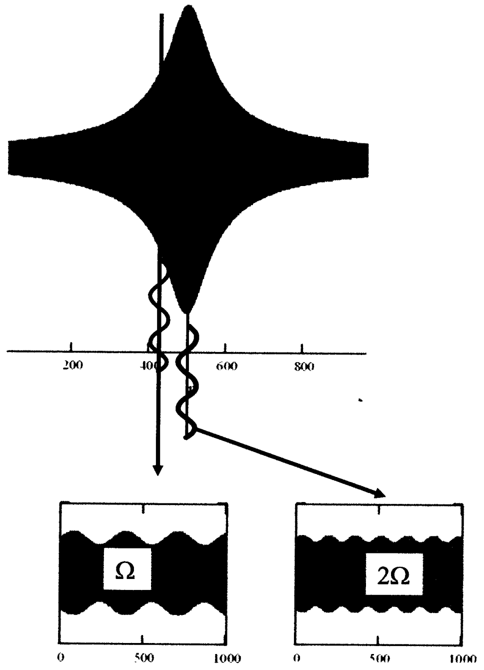


Figure 6. Quasi-static model for vibrato as the modulated output of the resonant response by a frequency-modulated excitation signal.

cies of the component partials relative to the resonances of the instrument and those of the surrounding acoustic and the positions of performer and listener in the performance space.

If we ignore dynamic effects, a weakly frequency-modulated ($a \ll 1$) sinusoidal force exciting an individual resonance will give a sinusoidal output modulated in amplitude at the vibrato frequency by an amount proportional to the slope of the resonance curve at the driving frequency, illustrated schematically in Figure 6. As the driving frequency approaches the resonant condition, the modulation at this frequency decreases to zero and is replaced by a smaller modulation component at double the modulation frequency, proportional to the curvature of the response curve.

For typical vibrato widths used by violinists, the broadening of the upper partials can easily exceed the average spacing between individual resonances, estimated by Cremer to be ∼45 Hz [20], as illustrated by the long-period FFT spectra in Figures 1 and 3. We then have to include the response from a number of relatively strongly excited structural resonances, superimposed on a slowly-varying response from all the other less-strongly excited normal modes of the instrument.

In Figure 7, we have simulated a typical region of the violin's response function for a set of random strength resonances spaced 45 Hz apart, with constant Q-values across the whole frequency range. The model is simply for qualitative illustration and is not intended to describe the resonant structure of a real violin. Increasing the Q-factors of the resonances will clearly increase the heights and slopes of the contributing resonance curves and hence the amplitude of the vibrato-induced fluctuation of the radiated sound.

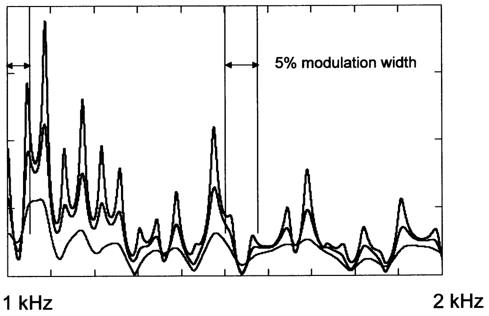


Figure 7. Simulated response curves for a violin with random strength resonances spaced 45 Hz apart for Q-values 10, 30 and 100. Modulation widths of 5% are shown by the pairs of vertical lines.

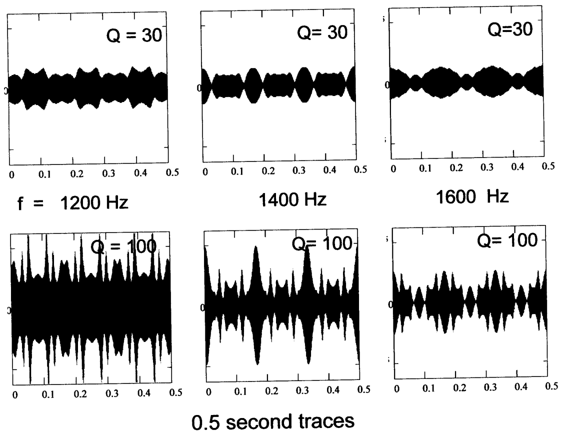


Figure 8. Vibrato induced fluctuations for three representative frequency modulated by $\pm 2.5\%$ at 6 Hz assuming a quasi-static response, illustrating the strong dependence on damping, but with a time-dependence which is symmetric with respect to time.

Figure 8 shows computed output fluctuations for three representative frequencies, for Q-values of 30 and 100 assuming a 5% vibrato-induced modulation width at 6 Hz. As anticipated, the amplitude and sharpness of the modulation features are very strongly dependent on the Q-values of the excited resonances. The complexity of the simulated envelopes is qualitatively similar to that observed for the sound of real instruments recorded close to the instrument. However, the fluctuations exhibit time-reversal symmetry, in contrast to the asymmetric fluctuations observed for real instruments played in real performing spaces.

To account for the asymmetry of the observed waveforms, we have to consider the dynamic response of both the violin and performance space. Dynamic effects will be significant for vibrato rates comparable with inverse

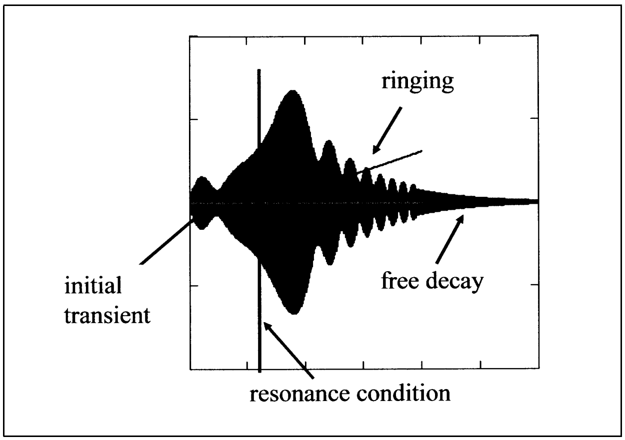
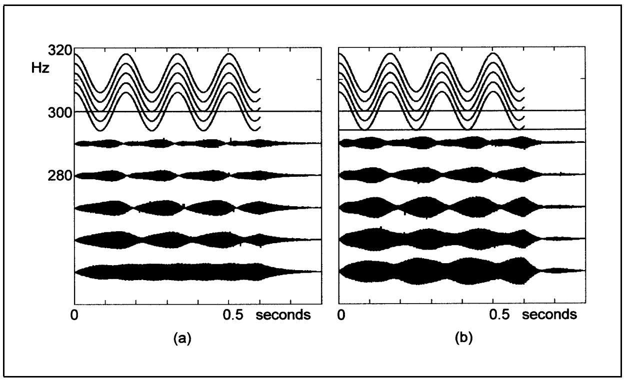


Figure 9. The dynamic response for an oscillator excited by a linearly increasing frequency sinusoidal input of constant amplitude passing through the resonance condition.



(a) (b)

Figure 10. (a) The response of a 300 Hz ($Q = 60$) resonator to a frequency modulated sine wave with a vibrato rate of 6 Hz and modulation parameter $a = 1$, for a sequence of inputs with mean frequencies shifted by successive fractional increments of 1%, as illustrated by the frequency/time curves; (b) the output for the excitation of two closely resonators with a fractional spacing of 2% of their natural frequencies.

damping times (i.e. $Q\Omega/\omega \geq 1/10\text{th}$) and for delay times τ from multiple reflections such that $Q\tau \geq 1/10\text{th}$.

A simple example of dynamic effects is illustrated in Figure 9, for a sinusoidal input with linearly increasing frequency passing through a single resonance. In contrast to the quasi-static response, there is an initial transient on turning on the excitation. The output then increases in amplitude as the resonant condition is approached. However, because of the delayed response, the maximum is not reached until the driving frequency has passed beyond the resonance condition. Thereafter, the amplitude of the output oscillates or "rings" with increasing frequency. This is a result of the resonator, once excited, continuing to ring at its natural frequency and beating with the resonator's instantaneous response at the driving frequency. Mathematically, this is equivalent to interference between the particular and complementary solutions of the forced simple harmonic oscillator equation. Once the excitation ceases, the oscillator decays at its natural resonance frequency.

Similar effects occur for frequency-modulated vibrato wave inputs. For an oscillator with resonant frequency ω_n excited by a phase-modulated sinusoidal input, the output can be expressed as the convolution of the oscillator impulse response and the frequency modulated input,

$$p(T) \sim \int_0^t \cos\big(\omega_n(t-t')\big)\mathrm{e}^{-(t-t')/\tau} \cdot \cos\big(\omega_n(1+\varepsilon) + a\cos\Omega t'\big)\mathrm{d}t', \qquad (4)$$

where τ is the decay time of the oscillator $(2Q/\omega_n)$ and ε is the fractional difference between the mean frequency of the excitation signal and the resonant frequency of the excited oscillator. The above result is easily generalised to include any number of excited resonators.

Figure 10a shows the computed fluctuations in amplitude of the output for a 300 Hz resonator with a Q-value of 60 excited by frequency-modulated (±5 Hz at a vibrato rate of 6 Hz) sinusoidal inputs, with fractional frequency shifts ε increasing by 1% between successive traces. When the mean excitation frequency coincides with that of the oscillator, the fluctuations in amplitude are relatively small. However, as the mean frequency of the input moves away from the resonant condition, the vibrato-induced fluctuations in amplitude increase dramatically, with asymmetrical waveforms passing through zero. When the excitation is switched off, the oscillator output decays at its natural vibration frequency.

The computed curves closely mimic the observed modulations of the partials of violin vibrato tones reported in the previous section. In particular, they reproduce the highly asymmetric modulation envelopes passing through zero in many cases. This behaviour can be qualitatively described as arising from the interference between the instantaneous excited output at the frequency modulated excitation and the freely decaying vibrations of the oscillator at its natural frequency.

If more than one oscillator is excited, additional structure occurs from beats between their natural frequencies, as illustrated in Figure 10b.

4. The synthesis of bowed vibrato tones

We now extend our dynamic model, to synthesise vibrato tones for any note played on a particular instrument, at any point in the performance space, from a single measurement of the tap-tone at the bridge recorded at the listener's position. The model assumes an ideal, frequency-modulated, sawtooth excitation force at the bridge.

The radiated sound $p(t, r, \theta, \phi)$ excited by a time-varying force $F(t)$ at the bridge can by expressed as the convolution

$$p(t, r, \theta, \phi) = \int_0^t F(t')p_0(t-t', r, \theta, \phi)\,\mathrm{d}t', \qquad (5)$$

where $p_0(t-t', r, \theta, \phi)$ is the transient response of the instrument for an impulsive δ-function force at the bridge recorded at the listener's position. $p_0(t-t', r, \theta, \phi)$ includes both the direct sound from the instrument and the reflections from the surrounding surfaces of the performance space. It can be derived from a single measurement of the impulsive response at the chosen position in the performance space, for a short impulse applied to the bridge in the bowing direction.

To a rather good approximation, the force $F(t)$ exerted on the bridge by the bowed string has a sawtooth waveform. In addition, there are additional small ripples, which vary from note to note and are critically dependent on the precise distance of the bow from the bridge (e.g. [17]). Because the perceived tone quality of a good instrument is remarkably uniform from note to note and is not critically dependent on the precise bow position for normal playing, such features are unlikely to be crucially important in defining the overall sound quality of an instrument. We therefore neglect the finer-scale structure of the bowed string waveform for the purpose of the present analysis. The bowed-string waveform will also have a slightly rounded discontinuity, as discussed in detail by Cremer [21], which can easily be incorporated into our model by applying a digital or analogue high-frequency cut-off filter to the simulated sound.

Our approach is not dissimilar to earlier electronic simulations of violin vibrato sounds by Fletcher and Sanders [2] and Mathews and Kohut [8]. These authors used electrical analogue models with a parallel bank of resonant LCR-circuits excited by a signal proportional to the measured force on the bridge, to simulate the sound of the bowed violin. In our model, we assume a simple sawtooth driving waveform and synthesise vibrato tones from the impulse response of the violin measured in a real performance space.

A saw-tooth waveform can be considered as a sequence of Helmholtz step-functions superimposed on an acoustically unimportant linearly rising (or falling) component, as illustrated in Figure 11. Mathematically, a step-function $H(t)$ can be considered as a continuing sequence of impulse functions, such that

$$H(t) = \int_0^t \delta(t')\,\mathrm{d}t', \tag{6}$$

so that for a Helmholtz step-function input equation (3) gives

$$p(t, r, \theta, \phi) = \int_0^t p_0(t', r, \theta, \phi)\,\mathrm{d}t'. \tag{7}$$

The step-function response can therefore be derived by integrating the sound from a single impulse. The sound of the bowed instrument played with vibrato can then be simulated by the superposition of a period-modulated sequence of derived step-function responses. We assume a constant bow speed. Changes in the period of the sawtooth waveform from the use of vibrato will therefore lead to small modulation of its amplitude. We can neglect such corrections, as they are very much smaller than the amplitude fluctuations induced by the frequency modulation.

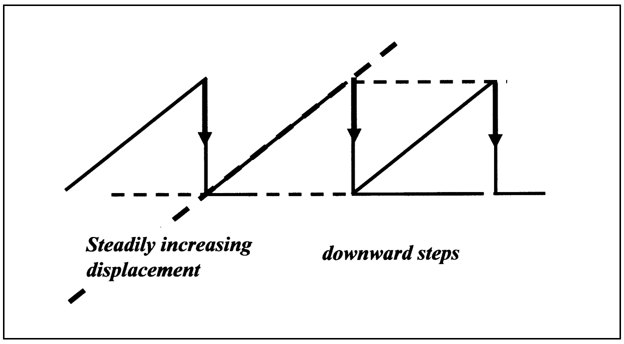


Figure 11. The bowed "sawtooth" force on the bridge considered as a sequence of downward Helmholtz step-functions superimposed on an acoustically unimportant linear ramp.

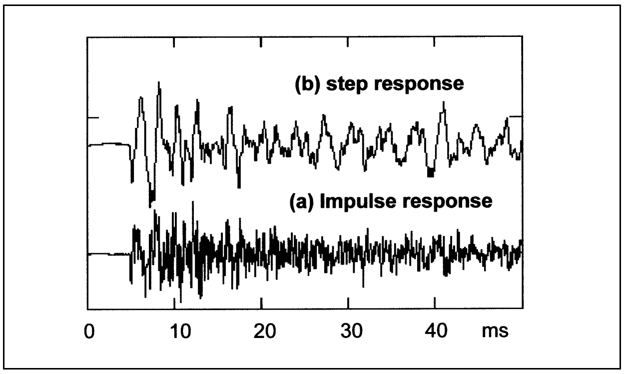


Figure 12. The first 50ms of the impulsive tap-tone response measured 2 m away from the violin and the derived Helmholtz step-function response.

The impulse response was determined by swinging a small pith-ball, 1.5 cm in diameter and 1.3 g in weight suspended on the end of a cotton thread, against the top of the G-string side of the bridge. The lower trace of Figure 12 shows the first 50 ms of the impulsive "tap-tone" of the Vuillaume violin, recorded at a distance of approximately 2 metres from the violin, in a small furnished room with dimensions $\sim 5 \times 6 \times 3\,\mathrm{m}^3$. The strings were heavily damped and the violin supported under the chin and held by the neck in the normal way. The upper trace in Figure 12 shows the computed step-function response, which accentuates the lower frequency components by the factor $1/\omega$, as is qualitatively evident by comparison of the traces.

The sounds of the impulse and step-function responses sound rather like the "tick" and "tock" of the traditional nursery-rhyme grandfather clock (SOUND 6 [12]).

The tap-tone and derived step-function responses are strongly dependent on the position of the listener or microphone relative to the instrument. This is illustrated in Figure 13 by tap-tones recorded 2 cm above the front plate of the instrument at the front edge of the chin-rest, close to the player's ear, and 2 m away from the violin (SOUND 7 [12]). In these measurements, the Vuillaume violin was held by the player in the usual way with the open-strings damped.

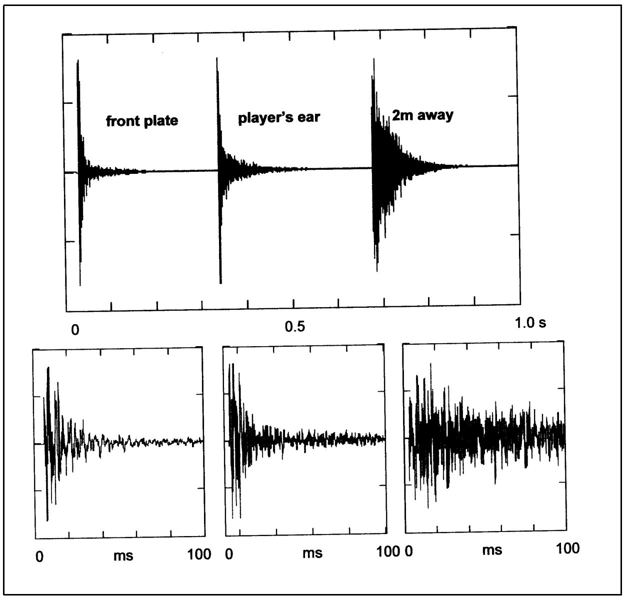


Figure 13. Tap tones at the bridge recorded just above the front plate of a Vuillaume violin, at the player's ear and 2m away in a small room, with damped violin strings, with expanded traces for the three tones below.

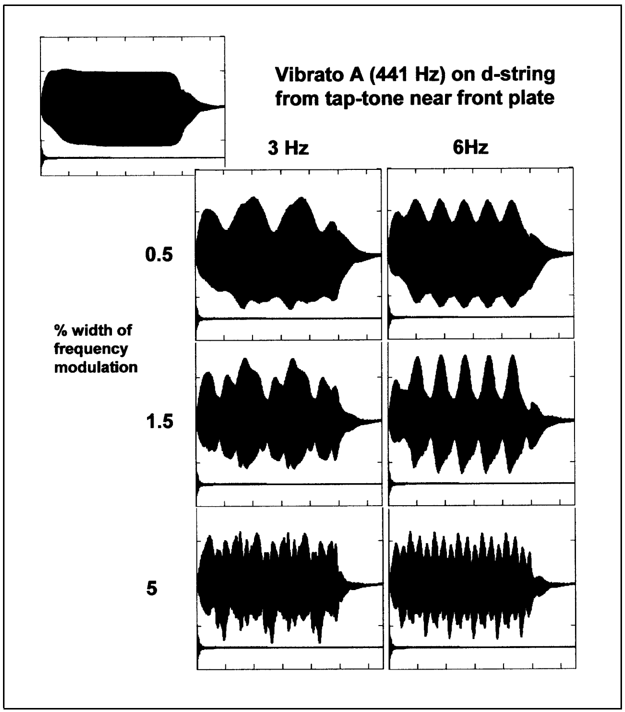


Figure 14. Simulated vibrato sounds generated by tap tones recorded close to the top plate of the Vuillaume violin, "played" with no vibrato (top left corner) and with the vibrato rates and % frequency modulation shown.

Close to the instrument the sound is dominated by the instrument and decays quite rapidly, whereas at a distance the sound is dominated by reflections from the walls of the room and dies away much less rapidly. The sound heard by the player is intermediate, with a significant contribution from both the violin and the surrounding acoustic, which provides very important feedback to the player on the quality of the sound being produced. Players strongly dislike playing in an anechoic chamber, the open-air, or a large concert hall with no nearby reflecting surfaces, where such feedback is largely absent.

To simulate the sound of the bowed violin, one simply superimposes a sequence of the derived step-function responses at the repetition rate of the bowed string. SOUND 8 [12] illustrates the sound at 2m, for repetition rates of 1 Hz, 10 Hz, and 200 Hz. The latter example simulates the "vibrato-less" sound of the bowed instrument. It has none of the fluctuations that characterise the sound as that of a violin.

To more closely mimic the sound of an instrument played with vibrato, we superimpose a sequence of derived step-functions, but now modulate the repetition frequency by a chosen fractional width and vibrato frequency. In addition, we modulate the input with a shaping function

$$(1 - \mathrm{e}^{-t/a})(1 - \mathrm{e}^{-(t-T)/b}),$$

where a and b are starting and ending time-constants, arbitrarily set to 25 ms, to simulate the initial attack and release of the bowed note after a time T.

Figure 14 shows synthesised waveform envelopes of bowed vibrato tones at 441 Hz as a function of vibrato rate and fractional frequency modulation, using tap-tones recorded close to the top plate of the Vuillaume violin, shown below each waveform. The envelope in the top left corner shows the characterless-sounding computed waveform without vibrato. When the period of the sawtooth input is modulated to simulate the use of vibrato, the waveform envelopes are strongly modulated and the sound closely resembles that of the real violin. Figure 15 shows similar data generated by tap-tones recorded at 2 m distance in a small room (SOUND 9 [12]). These examples illustrate the increasing complexity of the vibrato-fluctuations induced fluctuations with vibrato rate, width and distance from the violin.

The simulated sounds show that even fractional frequency vibrato widths as small as 0.5% produce very significant fluctuations in amplitude and "interest" to the simulated sounds. Simulations for other notes on the instrument showed very similar qualitative features, so there is nothing special about the particular choice of frequency illustrated.

5. Discussion

The above examples of real and simulated violin vibrato tones show that the dynamic response of both the violin and the surrounding acoustic are important in accounting for the complex fluctuations in amplitude of a violin played with vibrato. The importance of the dynamic response is underlined by the non-time-reversal symmetry of the fluctuations and their dependence on vibrato rate. Such features cannot be explained by quasi-static frequency modulation vibrato models.

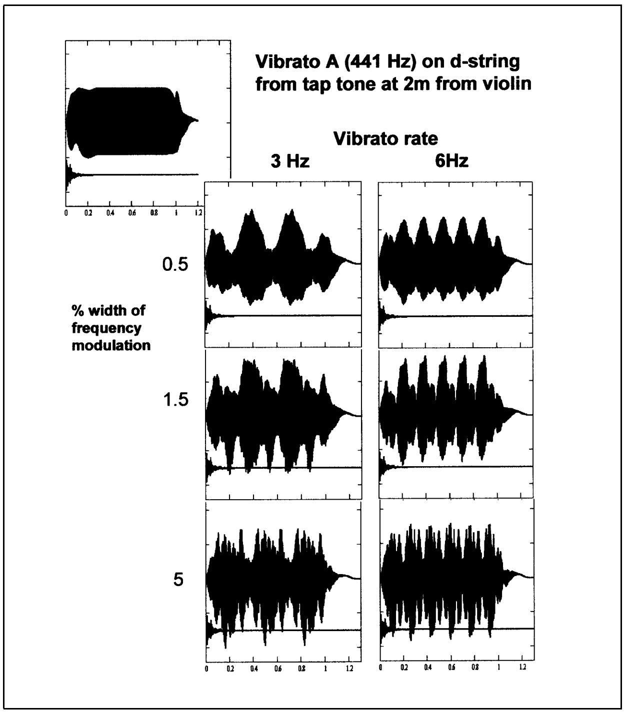


Figure 15. Simulated vibrato sounds of a Vuillaume violin generated by tap tones recorded 2m from the violin in a small room, "played" with no vibrato (top left corner) and with the vibrato rates and % frequency modulation shown.

We have demonstrated from both recorded and simulated violin tones that without vibrato (or other fluctuations) the sounds are indistinguishable from the bland and uninteresting sounds of a simple electronic synthesiser. It follows that vibrato- induced amplitude fluctuations are likely to play an important role in characterising the sound of long notes played on a violin and hence any subjective assessment of an instrument's quality. We intend to conduct listening tests on both real instruments played with varying amounts of vibrato and our computer synthesised vibrato tones with more precisely defined accurate vibrato widths and rates to assess the influence of vibrato on perceived quality.

Our measurements and synthesised tones show that room acoustic results in an increasing complexity in the fluctuations of the sound produced by an instrument played with vibrato. In contrast, the complexity of the fluctuations heard by the player is dominated by the intrinsic response of the violin itself. This helps to explain why the player finds it far easier to judge tonal differences in the quality of violins than a distant listener, especially in a resonant acoustic. Our model also explains the difficulty in characterising the intrinsic "quality" of an individual violin from commercial recordings, where the characteristic fluctuations in the violin vibrato sound can be dominated by the recording acoustic and unknown electronic "reverberation enhancements" by the sound engineer. In early acoustic recordings, the sound was recorded very close to the instrument, so that such effects were less important. The influence of the room acoustic in affecting the sound heard by the listener will also be an important factor in any subjective listening tests of violin quality, particularly for listeners at a distance from the violin in a resonant acoustic.

Weinreich [20] has drawn attention to the importance of the strongly frequency-dependent directionality of the violin's sound above 1 kHz in providing a "directional tone colour", which will vary from note to note and within notes played with vibrato. Our simulations, using the closely-recorded sound of the violin as an *isotropic* sound source in an artificial reverberant room, show that the use of vibrato also results in strongly directional and positional dependent fluctuations in the sound of a violin. This suggests that the spectral fluctuations between notes and within notes played with vibrato may depend just as strongly on the dynamic response of the violin and surrounding acoustic as on the directional properties of the violin. Such effects may therefore be equally important in contributing to what Weinreich refers to as the "sparkle" of the perceived sound of consecutive notes played in rapid succession and to the added "interest" of a note played with vibrato.

Finally, we briefly consider the influence of damping on violin vibrato sounds. Our dynamic model for vibrato shows that the complexity in the vibrato-induced amplitude fluctuations increases with increase in Q-values and "ringing" times of the structural resonances excited. If such fluctuations are important in the subjective assessment of a particular instrument, one might expect a close correlation between the damping and related Q-values of the instrument and its perceived quality. This is consistent with the generally held view that the highest quality "tone-wood", with a really good ringing quality, should be used for the acoustically important front and back plates of a violin.

Surprisingly, Curtin [22] has recently reported that the tap-tones of the plates of a number of fine Italian instruments are rather strongly damped. though this could also be associated with the lightness of the plates. It is also interesting to note that Marshall [23] and Bissinger [24], in modal analysis measurements, found that the damping of individual resonances below $\sim$1 kHz was significantly increased, when the violin was held under the chin in the normal way rather than supported by rubber bands. Any advantages of a violin carved from the highest quality tone-wood, would therefore appear to be negated, at least in part, by the player simply holding the instrument, though this could largely be a low frequency (below around 1 kHz) effect. It is also clear that, when the unbowed open-strings are free to vibrate, they will also contribute to the sound of the violin.

To confirm that both holding the violin and allowing the strings to vibrate freely have a significant affect on the sound of vibrato tones, we recorded tap-tones close to the top face of the Vuillaume violin in an anechoic space at a distance of 50 cm perpendicular to the front face of the violin. In these measurements, the violin was first freely supported by a rubber band with first damped then undamped strings. The measurements were then repeated with the vi-

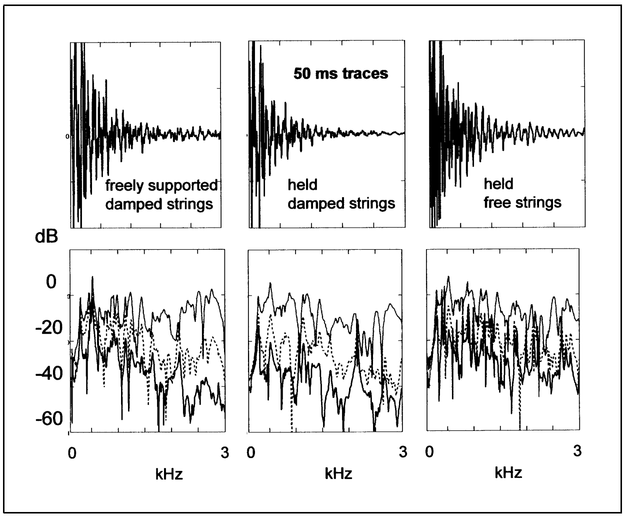


Figure 16. Tap tones at the bridge recorded close to the front plate of a Vuillaume violin illustrating the affect of holding the violin and the contributions to the sound from the undamped strings. The "un-windowed" long time FFT spectra shown are measured from the start of the waveform and at successive 12.5 ms delays, illustrating the damping of the various modes with time.

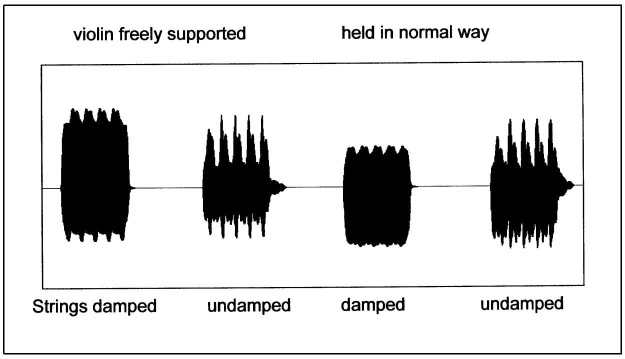


Figure 17. Simulated envelopes for a 441 Hz note with vibrato (±1.5% modulation width at 5 Hz), generated by tap-tones recorded at 50 cm for a Vuillaume violin freely suspended and supported in the normal way with open-strings first damped and then undamped.

olin supported under the chin and held at the neck in the normal way. Typical tap-tones for the Vuillaume violin are shown in Figure 16 and illustrated in SOUND10 [12]. The very different sound of the tap-tones, which would be used as generating functions for the synthesis of vibrato tones, is immediately apparent. The additional damping introduced by holding the violin is illustrated by the sequence of un-windowed, time-delayed, long-period FFT spectra shown below the tap tones. The sequence of FFTs derived from long period data sets starting from the beginning of the tap-tone and then with subsequent delays of 12.5 and 25 ms, during which times individual Fourier components will have decayed as $\sim e^{-t/\tau}$, where τ is the decay-time of the contributing modes.

These measurements confirm the large increase in damping of the modes when the instrument is held, especially below 1 kHz, apart from the Helmholtz air resonance, which is almost unchanged. However, when the strings are left free to vibrate, they more than compensate for the loss in signal from the structural resonances and are therefore likely to make a significant contribution to the amplitude fluctuations of any note played with vibrato, as illustrated by the simulated waveforms and sounds in Figure 17 and SOUND 11 [12]. A more detailed account of such measurements will be presented elsewhere.

The importance of the sympathetically excited open-string resonances can easily be demonstrated by comparing short bowed notes on an instrument with the unbowed strings first damped and then left free to vibrate – particularly for stopped notes that excite the partials of the open strings. When stopped there is very little after-sound from a short bowed note, which therefore sounds rather dead. In contrast, when the open strings are left free to vibrate, the sound continues to ring long after the note has been played giving a "bell-like" quality to notes. The effect of the sympathetic strings, which are impulsively excited by the bowed sawtooth force on the bridge and are coupled to acoustically radiating structural modes of the instrument, is not unlike the additional complexity introduced by the room acoustics. This could partly explain the preference of string players to play and composers to write "bright" music in "sharp-keys", particularly G, D and A, where interactions with the open strings and their partials give added warmth and interest to the sound. The use of additional sympathetic strings to enhance the sound of instruments was common in the pre-baroque period, as in bowed instruments like the Viola d'Amore.

6. Summary

In summary, we have demonstrated that:

1. vibrato is an important element in defining the sound of a violin, and is therefore likely to be important in any subjective assessment of its quality. Without fluctuations, the sound of a violin is indistinguishable from the output of a crude spectral synthesiser,
2. the partials of bowed notes on a violin played with vibrato exhibit very strong fluctuations in amplitude, which are asymmetrical with respect to time, depend on vibrato rate, and frequently pass through zero, reminiscent of beating effects between different frequency components,
3. the inverse period "frequency" departs significantly from the anticipated slowly varying cyclic modulation expected from the use of vibrato, especially in a resonant room acoustic,
4. the above effects can be described by a dynamic model for the frequency modulated multi-resonant response of the violin and performance acoustic,
5. the sound of a bowed violin note played with vibrato in a chosen performing acoustic can be simulated at any frequency, from a single measurement of the sound of a short tap at the bridge recorded at the position of the player or listener, assuming a frequency-modulated Helmholtz sawtooth bowing force at the bridge,

6. the fluctuations in sound of bowed notes played with vibrato will be very different for performer and listener, with a complexity in the amplitude fluctuations, which is increasingly dominated by the room acoustic at a distance from the violin. If, as we argue, such fluctuations are important in any assessment of the quality of a violin, this has important implications for subjective and quantitative listening tests,
7. because the observed and simulated fluctuations from the use of vibrato are largely associated with dynamic effects, the damping of the resonant modes of an instrument and the performance acoustic are of particular importance. We confirm earlier reports that holding the violin causes a significant increase in the damping of the vibrational modes of the violin, which is partly offset by the excitation of the less strongly damped partials of the undamped open-strings,
8. carefully designed listening tests are required to confirm the importance of the measured and predicted, dynamically-driven, large asymmetrical fluctuation on the perception of the vibrato tone of an instrument, and the extent to which the simulated model based on tap-tones simulates a realistic violin vibrato tone.

Acknowledgement

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